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Chapter 27

SINGLE-PHASE IM DESIGN

27.1 INTRODUCTION

By design we mean “dimensioning.” That is, the finding of a suitable geometry and manufacturing data and performance indexes for given specifications. Design, then, means first dimensioning (or synthesis), sizing, then performance assessment (analysis). Finally if the specifications are not met the process is repeated according to an adopted strategy until satisfactory performance is obtained.

On top of this, optimization is performed according to one or more objectives functions, as detailed in Chapter 18 in relation to three phase induction machines. Typical specifications (with a case study) are

- Rated power $P_n = 186.5 \text{ W}$ (1/4 HP)
- Rated voltage $V_{sn} = 115 \text{ V}$
- Rated frequency $f_{ln} = 60 \text{ Hz}$
- Rated power factor $\cos\phi_n = 0.98$ lagging service continuous or shortduty
- Breakdown p. u. torque 1.3-2.5
- Starting p. u. torque 0.5-3.5
- Starting p. u. current 5-6.5
- Capacitor p. u. maximum voltage 0.6-1.6

The breakdown torque p.u. may go as high as 4.0 for the dual capacitor configuration and special-service motors.

Also starting torques above 1.5 p. u. are obtained with a starting capacitor.

The split-phase IM is also capable of high starting and breakdown torques in p. u. as during starting both windings are active, at the expense of rather high resistance, both in the rotor and in the auxiliary stator winding.

For two (three) speed operation the 2 (3) speed levels in % of ideal synchronous speed have to be specified. They are to be obtained with tapped windings. In such a case it has to be verified that for each speed there is some torque reserve up to the breaking torque of that tapping.

Also for multispeed motors the locked rotor torque on low speed has to be less than the load torque at the desired low speed. For a fan load, at 50% as the low speed, the torque is 25 % and thus the locked rotor torque has to be less than 25%.

We start with the sizing of the magnetic circuit, move on to the selection of stator windings, continue with rotor slotting and cage sizing. The starting and (or) permanent capacitors are defined. Further on the parameter expressions are given and steady state performance is calculated. When optimization design is performed, objective (penalty) functions are calculated and constraints are

verified. If their demands are not met the whole process is repeated, according to a deterministic or stochastic optimization mathematical method, until sufficient convergence is reached.

27.2 SIZING THE STATOR MAGNETIC CIRCUIT

As already discussed in Chapter 14, when dealing with design principles of three phase induction machines, there are basically two design initiation constants, based on past experience

- The machine utilisation factor C_u in $\text{W/m}^3 D_0^2 L$
- The rotor tangential stress f_t in N/cm^2 or N/m^2

D_0 is the outer stator diameter and L -the stator stack length.

As design optimization methods advance and better materials are produced, C_u and f_t tend to improve slowly. Also, low service duty allows for improved in C_u and f_t .

However, in general, better efficiency requires larger C_u and lower f_t .

Figure 27.1 [1] presents standard data on C_u

$$C_u = D_0^2 L \quad (27.1)$$

for the three phase small power IMs.

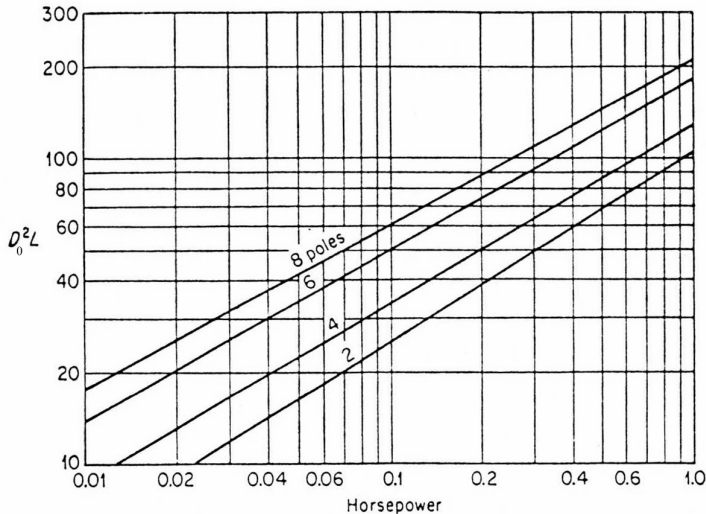


Figure 27.1 Machine utilization factor $C_u = D_0^2 L$ (cubic inches) for fractional/horsepower three-phase IMs.

Concerning the rated tangential stress there is not yet a history of its use but it is known that it increases with the stator interior (bare) diameter D_i , with values of around $f_t = 0.20 \text{ N/cm}^2$ for $D_i = 30 \text{ mm}$ to $f_t = 1 \text{ N/cm}^2$ for $D_i = 70 \text{ mm}$ or so, and more.

In general, any company could calculate $f_t(D_i)$ for the 2, 4, 6, 8 single phase IMs fabricated so far and then produce its own f_t database.

The rather small range of f_t variation may be exploited best in our era of computers for optimization design.

The ratio between stator interior (bore) diameter D_i and the external diameter D_o depends on the number of poles, on D_o and on the magnetic (flux densities) and electric (current density) loadings.

Figure 27.2. presents standard data [1] from three sources T. C. Lloyd, P. M. Trickey and Reference 2. In Reference 2, the ratio D_i/D_o is obtained for maximum airgap flux density in the airgap per given stator magnetisation m.m.f. in three phase IMs ($D_i/D_o = 0.58$ for $2p_1 = 2$, 0.65 for $2p_1 = 4$, 0.69 for $2p_1 = 6$, 0.72 for $2p_1 = 8$).

The D_i/D_o values of Reference 2 are slightly larger than those of P. M. Trickey, as they are obtained from a contemporary optimization design method for 3 phase IMs.

In our case study, from Figure 27.1, for 186.5 W (1/4 HP), $2p_1 = 4$ poles, we choose

$$C_u = D_o^2 L = 3.5615 \cdot 10^{-3} \text{ m}^3$$

with $L/D_o = 0.380$, $D_o = 0.137$ m, $L = 0.053$ m.

The outer stator punching diameter D_o might not be free to choose, as the frames for single phase IMs come into standardized sizes [3].

For 4 poles we choose from fig. 27.2 a kind of average value of the three sets of data $D_i/D_o = 0.60$. Consequently the stator bore diameter $D_i = 0.6 \times 0.137 = 82.8 \times 10^{-3}$ m.

The airgap $g = 0.3$ mm and thus the rotor external diameter $D_{or} = D_i - 2g = (82.8 - 2 \times 0.3) \times 10^{-3} = 82.2 \times 10^{-3}$ m.

The number of slots of stator N_s is chosen as for three phase IMs (the rules for most adequate combinations N_s and N_r established for three phase IMs hold in general also for single phase IMs see chapter 10). Let us consider $N_s = 36$ and $N_r = 30$.

The theoretical peak airgap flux density $B_g = 0.6-0.75$ T. Let us consider $B_g = 0.705$ T. Due to saturation it will be somewhat lower (flattened). Consequently, the flux per pole in the main winding Φ_m is

$$\Phi_m \approx \frac{2}{\pi} \cdot K_{dis} \cdot B_g \cdot \tau \cdot L \quad (27.3)$$

The pole pitch,

$$\tau = \pi \cdot \frac{D_i}{2p_1} = \frac{\pi}{4} \cdot 82.2 \times 10^{-3} \approx 64.57 \times 10^{-3} \text{ m} \quad (27.4)$$

With $K_{dis} = 0.9$

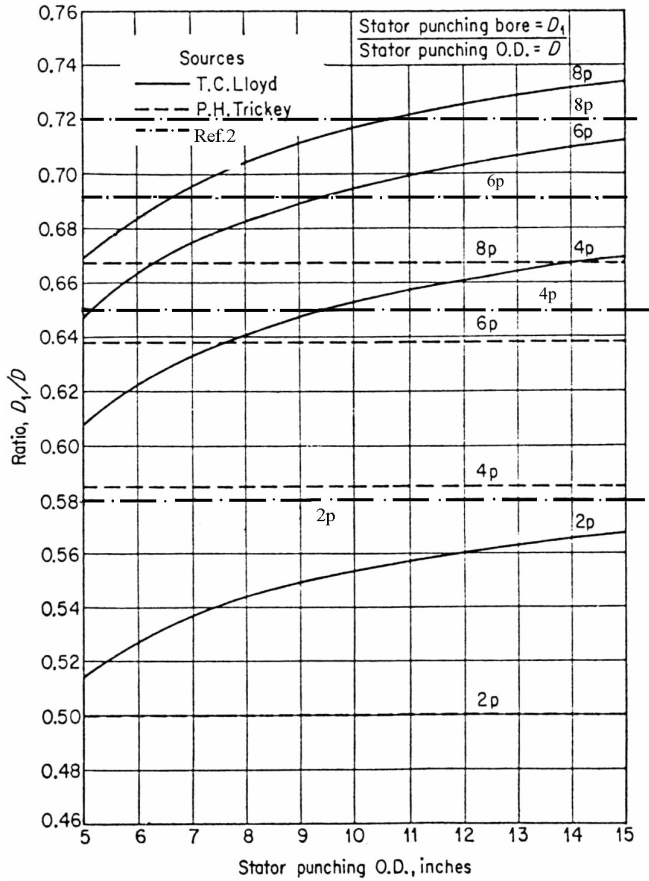


Figure 27.2 Interior/outer diameter ratio D_i/D_o versus D_o .

$$\Phi_m = \frac{2}{\pi} 0.9 \cdot 0.705 \cdot 64.57 \times 10^{-3} \cdot 0.053 \approx 1.382 \times 10^{-3} \text{ Wb}$$

Once we choose the design stator back iron flux density $B_{cs} = 1.3\text{-}1.7 \text{ T}$, the back iron height h_{cs} may be computed from

$$h_{cs} = \frac{\Phi_m}{2 \cdot B_{cs} \cdot L} = \frac{1.382 \times 10^{-3}}{2 \cdot 1.5 \times 0.053} = 8.69 \times 10^{-3} \approx 9 \times 10^{-3} \text{ m} \quad (27.4)$$

The stator slot geometry is shown on [Figure 27.3](#).

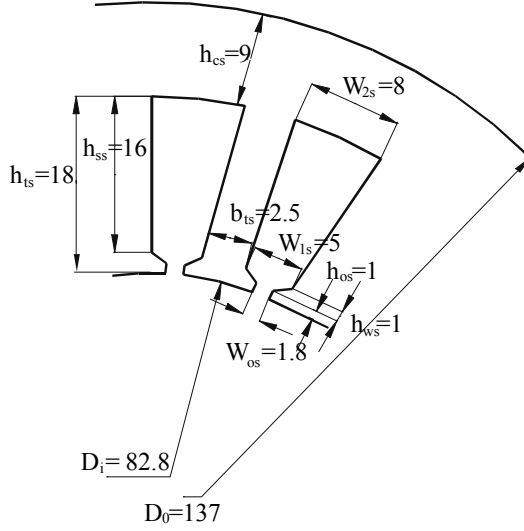


Figure 27.3 Stator slot geometry in mm

The number of slots per pole is $N_s/2p_1 = 36/(2 \times 2) = 9$. So the tooth width b_{ts} is

$$b_{ts} = \frac{\Phi_m \cdot 2p_1}{N_s \cdot B_{ts} \cdot L} \quad (27.5)$$

With the tooth flux density $B_{ts} \cong (0.8-1.0) B_{cs}$

$$b_{ts} = \frac{1.382 \times 10^{-3} \times 4}{36 \cdot 1.3 \cdot 0.053} \approx 2.55 \times 10^{-3} \text{ m} \quad (27.6)$$

This value is close to the lowest limit in terms of punching capabilities.

Let us consider $h_{os} = 1 \times 10^{-3} \text{ m}$, $w_{os} = 6g = 6 \times 0.3 \times 10^{-3} = 1.8 \times 10^{-3} \text{ m}$.

Now the lower and upper slot width w_{1s} and w_{2s} are

$$W_{1s} = \frac{\pi(D_i + 2(h_{os} + h_{ws}))}{N_s} - b_{ts} = \left(\frac{\pi(82.8 + 2(1 + 1))}{36} - 2.55 \right) \times 10^{-3} \quad (27.7)$$

$$\approx 5.00 \times 10^{-3} \text{ m}$$

$$W_{2s} = \frac{\pi(D_o - 2h_{cs})}{N_s} - b_{ts} = \left(\frac{\pi(137 - 2 \times 9)}{36} - 2.5 \right) \times 10^{-3} \approx 8.00 \times 10^{-3} \text{ m} \quad (27.8)$$

The useful slot height is

$$h_{ts} = \frac{D_o}{2} - h_{cs} - h_{ws} - h_{os} - \frac{D_i}{2} = \left(\frac{137}{2} - 9 - 2 - \frac{82.8}{2} \right) \times 10^{-3} \quad (27.9)$$

$$\approx 16.0 \times 10^{-3} \text{ m}$$

So the “active” stator slot area A_s is

$$A_s = \frac{(W_{1s} + W_{2s})}{2} h_{ss} = \frac{(5.00 + 8.00)}{2} \times 16 \times 10^{-6} = 104 \times 10^{-6} \text{ m}^2 \quad (27.9)$$

For slots which host both windings or in split phase IMs some slots may be larger than others.

27.3 SIZING THE ROTOR MAGNETIC CIRCUIT

The rotor slots for single phase IMs are either round or trapezoidal or in between (Figure 27.4).

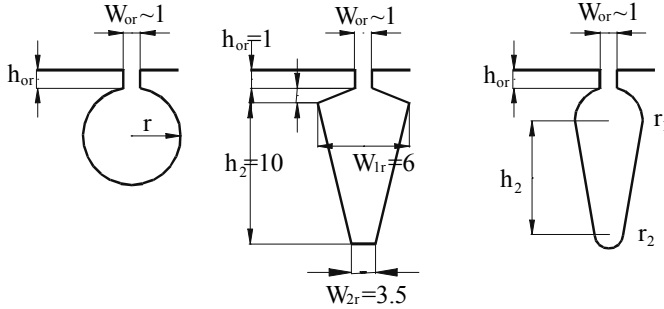


Figure 27.4 Typical rotor slot geometries

With 30 rotor slots, the rotor slot pitch τ_{sr} is

$$\tau_{sr} = \frac{\pi \cdot D_r}{N_r} = \frac{\pi \cdot 82.2 \times 10^{-3} \cdot 4}{30} = 8.6 \times 10^{-3} \quad (27.10)$$

With a rotor tooth width $b_{tr} = 2.6 \times 10^{-3}$ m the tooth flux density B_{tr} is

$$B_{tr} = \frac{\Phi \cdot 2p_1}{N_r \cdot b_{tr} \cdot L} = \frac{1.388 \times 10^{-3} \cdot 4}{30 \cdot 2.6 \cdot 10^{-3} \cdot 0.053} = 1.343 \text{ T} \quad (27.11)$$

The rotor slots useful area is in many cases (35–60) % of that of the stator slot

$$A_r = 0.38 \frac{A_s \cdot N_s}{N_r} = 0.38 \times 104 \times 10^{-6} \times \frac{36}{30} = 47.42 \times 10^{-6} \text{ m}^2 \quad (27.12)$$

The trapezoidal rotor slot (Figure 27.4), with

$$\begin{aligned} h_{or} &= W_{or} = 1 \times 10^{-3} \text{ m}; h_{or1} = 1 \times 10^{-3} \text{ m} \\ \text{has } W_{lr} &\approx \tau_{sr} - b_{tr} = (8.6 - 2.6) \times 10^{-3} = 6 \times 10^{-3} \text{ m} \end{aligned} \quad (27.13)$$

Adopting a slot height $h_2 = 10 \times 10^{-3} \text{ m}$ we may calculate the rotor slot bottom width W_{2r}

$$W_{2r} = \frac{2A_r}{h_2} - W_{ls} = \frac{2 \cdot 47.40 \times 10^{-3}}{10 \times 10^{-3}} - 6 \times 10^{-3} = 3.48 \times 10^{-3} \text{ m} \quad (27.14)$$

A “geometrical” verification is now required.

$$\begin{aligned} W_{2r} &= \frac{\pi[D_r - (h_{or} + h_{or1} + h_2) \cdot 2]}{N_r} - b_{tr} = \\ &= \frac{\pi[82.2 - 2(1 + 1 + 10)] \times 10^{-3}}{30} - 2.6 \times 10^{-3} = 3.49 \times 10^{-3} \text{ m} \end{aligned} \quad (27.15)$$

As (27.14) and (27.15) produce the same value of slot bottom width, the slot height h_2 has been chosen correctly. Otherwise h_2 should have been changed until the two values of W_{2r} converged.

To avoid such an iterative computation (27.14)–(27.15) could be combined into a second order equation with h_2 as the unknown after the elimination of W_{2r} .

A round slot with a diameter $d_r = W_{2r} = 6 \times 10^{-3} \text{ m}$ would have produced an area $A_r = \frac{\pi}{4} (6 \times 10^{-3})^2 = 28.26 \times 10^{-6} \text{ m}^2$.

This would have been too small a value unless copper bars are used instead of aluminum bars.

The end ring area A_{ring} is

$$A_{ring} \approx A_r \times \frac{1}{2 \sin \frac{\pi \cdot p_1}{N_r}} = 47.42 \times 10^{-6} \frac{1}{2 \sin \frac{\pi \cdot 2}{30}} = 114 \times 10^{-6} \text{ m}^2 \quad (27.16)$$

With a radial height $b_r = 15 \times 10^{-3} \text{ m}$, the ring axial length $a_r = \frac{A_{ring}}{b_r} = 7.6 \times 10^{-3} \text{ m}$.

27.4 SIZING THE STATOR WINDINGS

By now the number of slots in the stator is known $N_s = 36$. As we do have a permanent capacitor motor the auxiliary winding is always operational. It thus seems natural to allocate both windings about same number of slots in fact 20/16. In general single layer windings with concentrated coils are used. ([Figure 27.5](#))

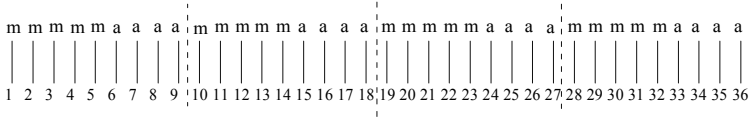


Figure 27.5 Stator winding m-main phase; a-auxiliary phase.

With identical coils per slot and phase; the winding factors for the two phases (Chapter 4) are

$$K_{wm} = \frac{\sin\left(q_m \frac{\alpha_s}{2}\right)}{q_m \sin \frac{\alpha_s}{2}} = \frac{\sin\left(5 \times \frac{\pi}{36}\right)}{5 \sin \frac{\pi}{36}} = 0.9698$$

$$K_{wa} = \frac{\sin\left(q_a \frac{\alpha_s}{2}\right)}{q_a \sin \frac{\alpha_s}{2}} = \frac{\sin\left(4 \times \frac{\pi}{36}\right)}{4 \sin \frac{\pi}{36}} = 0.9810 \quad (27.17)$$

There are 5 ($q_m = 5$) slots per pole per phase for the main winding and 4 ($q_a = 4$) for the auxiliary one.

In case the number of turns/phase in various slots is not the same, the winding factor can be calculated as shown below.

As detailed in chapter 4, two types of sinusoidal windings may be built (Figure 27.6)

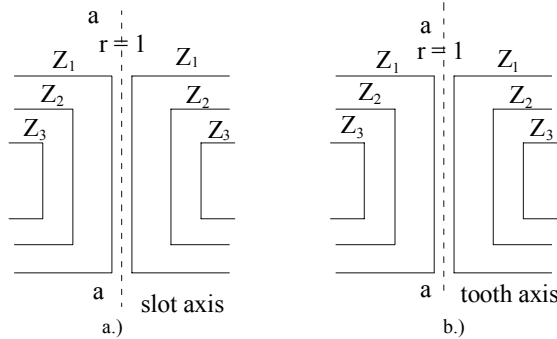


Figure 27.6 Sinusoidal windings a.) with slot axis symmetry; b.) with tooth axis symmetry.

When the total number of slots per pole per phase is an odd number, concentrated coils with slot axis symmetry seem adequate. In contrast, for even number of slots/pole/phase, concentrated coils with tooth axis symmetry are recommended.

Should we have used such windings for our case with $q_m = 5$ and $q_a = 4$, slot axis symmetry would have applied to the main winding and tooth axis symmetry to the auxiliary winding. A typical coil group is shown on Figure 27.7.

From Figure 27.7 the angle between the axis a-a (Figure 27.6-27.7) and the k^{th} slot, β_{kv} (for the v^{th} harmonic), is

$$\beta_{kv} = \gamma \frac{\pi}{N_s} v + (k-1) \frac{2\pi v}{N_s} \quad (27.18)$$

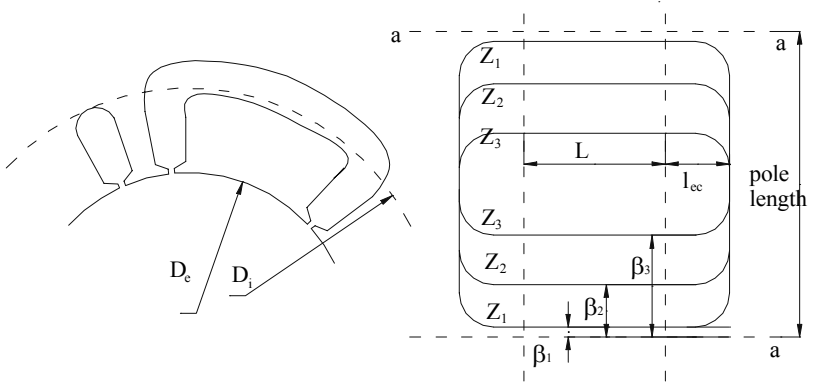


Figure 27.7 Typical “overlapping” winding

The effective number of conductors per half a pole Z'_v is [4]

$$Z'_v = \sum_{k=1}^n Z_k \cos \beta_{kv} \quad (27.19)$$

$$N_v = 4p_1 \cdot Z'_v$$

N_v is the effective number of conductors per phase.

The winding factor K_{wv} is simply

$$K_{wv} = \frac{Z'_v}{\sum_{k=1}^n Z_k} = \frac{\sum_{k=1}^n Z_k \cos \beta_{kv}}{\sum_{k=1}^n Z_k} \quad (27.20)$$

The length of the conductors per half a pole group of coil, l_{cn} , is [3]

$$l_{cn} \approx \sum_{k=1}^n Z_k \left[L + (\pi - 2)L_{ec} + D_e \left(\frac{\pi}{2p_1} - \beta_{kl} \right) \right] \quad (27.21)$$

So the resistance per phase R_{phase} is

$$R_{\text{phase}} = \rho_{Co} \frac{4p_1 l_{cn}}{A_{con}} \quad (27.22)$$

A_{con} is the conductor (magnet wire) cross-section area. A_{con} depends on the total number of conductors per phase, slot area and design current density.

The problem is that, though the supply current at rated load may be calculated as

$$I_s = \frac{P_n}{\eta_n V_s \cos \varphi} = \frac{186.5}{0.7 \times 115 \times 0.98} = 2.36 \text{ A} , \quad (27.23)$$

with an assigned value for the rated efficiency, η_n , still the rated current for the main and auxiliary currents I_m , I_a are not known at this stage.

I_m and I_a should be almost 90° phase shifted for rated load and

$$I_s = I_m + I_a \quad (27.24)$$

Now as the ratio $a = \frac{I_m}{I_a} = \frac{N_a}{N_m}$, for symmetry, we can assume that

$$I_m \approx \frac{I_s}{\sqrt{1 + \left(\frac{1}{a}\right)^2}} \quad (27.25)$$

With a the turns ratio in the interval $a = 1.0-2$, in general $I_m = I_s \cdot (0.700 - 0.90)$.

The number of turns of the main winding N_m is to be determined by observing that the e.m.f. in the main winding $E_m = (0.96 - 0.98) \cdot V_s$ and thus with (27.3)

$$E_m = \pi \sqrt{2} \Phi_m N_m k_{wm} f_{1n} \quad (27.26)$$

Finally

$$N_m = \frac{0.97 \times 115}{\pi \sqrt{2} 21.382 \times 10^{-3} \times 0.9698 \times 60} = 313 \text{ turns}$$

In our case, the main winding has 10 ($p_1 q_m = 2 \times 5 = 10$) identical coils. So the number of turns per slot n_{sm} is

$$n_{sm} = \frac{N_m}{q_m p_1} = \frac{313}{5 \times 2} = 31 \text{ turns / coil} \quad (27.27)$$

Assuming the turn ratio is $a = 1.5$, the number of turns per coil in the auxiliary winding is

$$n_{sa} = \frac{N_m \times a \times k_{wm}}{q_a \times p_1 \times k_{wa}} = \frac{310 \times 1.5 \times 0.9698}{4 \times 2 \times 0.9810} \approx 57 \text{ turns / coil} \quad (27.28)$$

As the slots are identical and their useful area is (from 27.9) $A_s = 104 \times 10^{-6} \text{ m}^2$, the diameters of the magnetic wire used in two windings are

$$\begin{aligned} d_m &= \sqrt{\frac{4 \cdot A_s K_{\text{fill}}}{\pi n_{\text{sm}}}} = 10^{-3} \sqrt{\frac{4 \cdot 104 \times 0.4}{3.14 \cdot 31}} \approx 1.3 \times 10^{-3} \text{ m} \\ d_a &= \sqrt{\frac{4 \cdot A_s K_{\text{fill}}}{\pi n_{\text{sa}}}} = 10^{-3} \sqrt{\frac{4 \cdot 104 \times 0.4}{3.14 \cdot 57}} \approx 1.0 \times 10^{-3} \text{ m} \end{aligned} \quad (27.29)$$

The filling factor was considered rather large $K_{\text{fill}} = 0.4$.

The predicted current density in the two windings would be

$$\begin{aligned} j_{\text{Com}} &= \frac{I_m}{\frac{\pi d_m^2}{4}} = \frac{2.36}{\left(\sqrt{1 + \left(\frac{1}{1.5} \right)^2} \right) \cdot \pi \cdot \frac{1.3^2}{4} \cdot 10^{-6}} = 1.4847 \times 10^6 \text{ A/m}^2 \\ j_{\text{Coa}} &= \frac{I_m}{\frac{a \cdot \pi \cdot d_a^2}{4}} = \frac{1.9696}{\frac{1.5 \times \pi \cdot 1^2 \times 10^{-6}}{4}} = 1.6726 \times 10^6 \text{ A/m}^2 \end{aligned} \quad (27.30)$$

The forecasted current densities, for rated power, are rather small so good efficiency (for the low power considered here) is expected.

An initial value for the permanent capacitor has to be chosen.

Let us assume complete symmetry for rated load, with

$$I_{\text{an}} = \frac{I_{\text{mn}}}{a} = \frac{1.96}{1.5} = 1.3 \text{ A}.$$

Further on

$$V_{\text{an}} = V_{\text{mn}} \cdot a = 115 \times 1.5 = 172 \text{ V} \quad (27.31)$$

The capacitor voltage V_c is

$$V_c = \sqrt{V_s^2 + V_{\text{an}}^2} = 115 \sqrt{1 + 1.5^2} = 207.3 \text{ V} \quad (27.32)$$

Finally the permanent capacitor C is

$$C = \frac{I_a}{\omega_1 V_c} = \frac{1.3}{2 \cdot \pi \cdot 60 \cdot 207.3} \approx 16 \mu\text{F} \quad (27.33)$$

We may choose a higher value of C than 16 μF (say 25 μF) to increase somewhat the breakdown and the starting torque.

27.5 RESISTANCES AND LEAKAGE REACTANCES

The main and auxiliary winding resistances R_{sm} and R_{sa} (27.22) are

$$R_{sm} = \rho_{Co} \cdot 4 \cdot \pi \frac{l_{cnm}}{\frac{\pi d_m^2}{4}}$$

$$R_{sa} = \rho_{Co} \cdot 4 \cdot \pi \frac{l_{cna}}{\frac{\pi d_a^2}{4}} \quad (27.34)$$

The length of the coils/half a pole for main and auxiliary windings l_{cnm} (27.21) are

$$l_{cnm} \approx n_{cm} \left[\left(2 + \frac{1}{2} \right) L + 3(\pi - 2)L_{ec} + D_e \left(\frac{3\pi}{2p_1} - \frac{2\pi}{36} - \frac{4\pi}{36} \right) \right] =$$

$$= 31 \times \left[\left(2 + \frac{1}{2} \right) \times 0.053 + 3(\pi - 2) \times 0.025 + 0.14 \left(\frac{3\pi}{4} - \frac{2\pi}{36} - \frac{4\pi}{36} \right) \right] = 9.805m \quad (27.35)$$

$$l_{cna} \approx n_{ca} \left[2L + 3(\pi - 2)L_{ec} + D_e \left(\frac{2\pi}{2p_1} - \frac{2\pi}{36} - \frac{4\pi}{36} \right) \right] =$$

$$= 57 \times \left[2 \times 0.053 + 3(\pi - 2) \times 0.025 + 0.14 \left(\frac{2\pi}{4} - \frac{\pi}{18} - \frac{\pi}{9} \right) \right] = 11.38m \quad (27.36)$$

Finally

$$R_{sm} = \frac{2.1 \times 10^{-8} \times 4 \times 2 \times 9.805}{\frac{\pi}{4} \times 1.3^2 \times 10^{-6}} = 1.2417\Omega$$

$$R_{sa} = \frac{2.1 \times 10^{-8} \times 4 \times 2 \times 11.38}{\frac{\pi}{4} \times 1^2 \times 10^{-6}} = 2.435\Omega$$

The main winding leakage reactance X_{sm}

$$X_{sm} = X_{ss} + X_{se} + X_{sd} + X_{skew} \quad (27.37)$$

where X_{ss} -the stator slot leakage reactance
 X_{se} -the stator end connection leakage reactance
 X_{sd} -the stator differential leakage
 X_{skew} -skewing reactance

$$X_{se} + X_{ss} = 2\mu_0\omega_1 \frac{W_l^2 \cdot L}{p \cdot q} (\lambda_{ss} + \lambda_{se}) \quad (27.38)$$

with (chapter 6 and [Figure 27.3](#))

$$\lambda_{ss} = \frac{2h_{ss}}{3(W_{1s} + W_{2s})} + \frac{2h_{ws}}{W_{1s} + W_{2s}} + \frac{h_{os}}{W_{os}} = \frac{2 \times 16}{3(5+8)} + \frac{2 \times 1}{5+8} + \frac{1}{1.8} = 1.53 \quad (27.39)$$

Also (chapter 6, equation (6.28))

$$\lambda_{se} = \frac{0.67 \cdot \frac{q_m}{2}}{L} (l_{ec} - 0.64 \cdot \tau) \quad (27.40)$$

It is $q_m/2$ as the q coil/pole are divided in two sections whose end connections go in opposite directions (Figure 27.5).

The end connection average length l_{ec} is

$$l_{ec}^m = \frac{l_{cn} \times 2}{q_m n_{cm}} - L \quad (27.41)$$

With l_{cn} from (27.21)

$$l_{ec}^m = \frac{9.805 \times 2}{5 \times 31} - 0.053 = 0.0735m$$

$$\lambda_{se} = \frac{0.67 \times \frac{5}{2}}{0.053} (0.0735 - 0.64 \times 0.065) = 1.008$$

The differential leakage reactance X_{sd} (chapter 6, Equation (6.2) and (6.9) and (Figure 6.2-Figure 6.5)) is

$$\frac{X_{sd}}{X_{mm}} = \sigma_{ds} \cdot \Delta_d = 2.6 \times 10^{-2} \times 0.92 = 2.392 \times 10^{-2} \quad (27.42)$$

for $q_m = 5$, $N_s = 36$, $N_r = 30$, one slot pitch skewing of rotor.

X_{mm} is the nonsaturated magnetization reactance of the main winding (Chapter 5, Equation (5.115))

$$\begin{aligned} X_{mm} &= \omega_l \frac{4\mu_0}{\pi^2} (W_m \cdot K_{wml})^2 \frac{L \cdot \tau}{p_l \cdot g \cdot K_c} = \\ &= 2\pi \cdot 60 \frac{4 \cdot 4\pi \times 10^{-7}}{\pi^2} (10 \times 31 \times 0.9698)^2 \frac{0.053 \cdot 0.065}{2 \cdot 0.3 \times 10^{-3} \cdot 1.3} = 72.085\Omega \end{aligned}$$

K_c -Carter coefficient; g -the airgap.

$$\frac{X_{skew}}{X_{mm}} = (1 - K_{skew}^2) = 1 - \frac{\sin^2 \frac{\alpha_{skew}}{2}}{(\alpha_{skew})^2} = 1 - \frac{\sin^2 \frac{\pi}{18}}{\left(\frac{\pi}{18}\right)^2} = 9.108 \times 10^{-3} \quad (27.43)$$

Finally the main winding leakage reactance X_{sm} is

$$X_{sm} = 2 \times 4\pi \times 10^{-7} \frac{2\pi \cdot 60 \times 310^2 \cdot 0.053 \cdot (1.53 + 1.008)}{2 \times 5} + 72.085 \cdot (23.92 + 9.108) \cdot 10^{-3} = 4.253\Omega$$

The auxiliary winding is placed in identical slots and thus the computation process is similar. A simplified expression for X_{sa} in this case is

$$X_{sa} \approx X_{sm} \left(\frac{W_a}{W_m} \right) = 4.253 \times \left(\frac{57 \times 8}{31 \times 10} \right)^2 = 9.204\Omega \quad (27.44)$$

The rotor leakage reactance, after its reduction to the main winding (chapter 6, equations (6.86)-(6.87)) is

$$X_{rm} = X_{rmd} + X'_{bem} = \omega_1 (L_{rmd} + L'_{bem}) \quad (27.45)$$

The rotor-skewing component has been “attached” to the stator and the zig-zag component is lumped into differential one X_{rmd} .

From Equation (6.16) (Chapter 6)

$$X_{rmd} = \sigma_{dr0} X_{mm} = 2.8 \times 10^{-2} X_{mm} \quad (27.46)$$

With $N_r/p_1 = 30/2 = 15$ and one slot pitch skewing, from [Figure 6.4](#)

$$\sigma_{dr0} = 2.8 \times 10^{-2}$$

The equivalent bar-end ring leakage inductance L'_{bem} is (Equation 6.86)

$$L'_{bem} = L_{bem} \frac{12 \cdot K_{wm}^2 \cdot W_m^2}{N_r} \quad (27.47)$$

$$\text{From (6.91)-(6.92)} \quad L_{ben} = \mu_0 l_b \lambda_b + 2\mu_0 \lambda_{ei} \cdot 2\pi \cdot \frac{D_{ir}}{N_r} \quad (27.48)$$

With the rotor bar (slot) and end ring permeance coefficients, λ_b and λ_{ei} (from (6.18) and (6.46) and [Figure 27.5](#))

$$\lambda_b = \frac{h_{or}}{W_{or}} + \frac{2h_{or1}}{(W_{o1} + W_{o2})} + \frac{2h_2}{3(W_{1r} + W_{2r})} = \frac{1}{1} + \frac{1 \times 2}{1 + 6} + \frac{2 \times 10}{3(6 + 3.5)} = 1.987 \quad (27.49)$$

$$\lambda_{ei} = \frac{2.3 \times D_{ir}}{4 \cdot N_r \cdot L \cdot \sin^2 \left(\frac{\pi p_1}{N_r} \right)} \log \frac{4.7 \cdot D_{ir}}{a_r + 2 \cdot b_r} \quad (27.50)$$

With $D_{ir} \approx D_r - b = (82.8 - 15) \times 10^{-3} = 67.8 \times 10^{-3} \text{ m}$, $a_r = 7.6 \times 10^{-3} \text{ m}$ and $b_r = 15 \times 10^{-3} \text{ m}$ from (27.16)

$$\lambda_{ei} = \frac{2.3 \times 67.8 \times 10^{-3}}{4 \cdot 30 \cdot 0.053 \cdot \sin^2\left(\frac{\pi}{15}\right)} \log\left(\frac{4.7 \times 67.8}{2 \times 15 + 7.6}\right) = 0.579$$

so from (27.48)

$$L_{ben} = 1.256 \times 10^{-6} \left(0.053 \times 1.987 + 0.579 \frac{2\pi \times 67.8 \times 10^{-3}}{30} \right) = 0.1425 \times 10^{-6} \text{ H}$$

Now from (27.45) and (24.76)

$$\begin{aligned} X_{rm} &= 2.8 \times 10^{-2} X_{mm} + \omega_l L_{ben} \frac{12K_{wm}^2 W_m^2}{N_r} = \\ &= 2.8 \times 10^{-2} \times 72.085 + 2\pi \times 60 \times 0.1425 \times 10^{-6} \frac{12 \times 0.9698^2 \times 310^2}{30} = \end{aligned} \quad (27.51)$$

$$= 3.961 \Omega$$

The rotor cage resistance

$$R_{rm} = R_{be} \frac{12K_{wm}^2 W_m^2}{N_r} \quad (27.52)$$

With R_{be} (equation (6.63))

$$R_{be} = R_b + \frac{R_{ring}}{2 \cdot \sin^2\left(\frac{\pi \cdot p_l}{N_r}\right)} \quad (27.53)$$

$$\begin{aligned} R_b &= \rho_{Co} \frac{l_b}{A_r} = 3 \cdot 10^{-8} \frac{0.065}{47.42 \times 10^{-6}} = 4.112 \times 10^{-5} \Omega \\ R_{ring} &= \rho_b \frac{l_{ring}}{A_{ring}} = 3 \times 10^{-8} \frac{2\pi \times 67.8 \times 10^{-3}}{1.14 \times 10^{-4} \times 30} = 3.735 \times 10^{-6} \Omega \end{aligned} \quad (27.54)$$

Finally

$$R_{rm} = \left(4.112 \times 10^{-5} + \frac{3.735 \times 10^{-6}}{2 \cdot \sin^2 \frac{\pi \cdot 2}{30}} \right) \cdot 12 \times \frac{0.9698^2 \cdot 310^2}{30} = 3.048 \Omega$$

Note The rotor resistance and leakage reactance calculated above are not affected by the skin effects.

27.6 THE MAGNETIZATION REACTANCE X_{mm}

The magnetisation reactance X_{mm} is affected by magnetic saturation which is dependent on the resultant magnetisation current I_m .

Due to the symmetry of the magnetisation circuit it is sufficient to calculate the functional $X_m(I_m)$ for the case with current in the main winding ($I_a = 0$, bare rotor).

The computation of $X_m(I_m)$ may be performed analytically (see Chapter 6 or Ref. 4) or by F.E.M. For the present case, to avoid lengthy calculations let us consider X_{mm} constant for a moderate saturation level ($1+K_s = 1.5$)

$$X_{mm} = (X_{mm})_{unsat} \frac{1}{1 + K_s} = \frac{72.085}{1.5} = 48.065\Omega \quad (27.55)$$

27.7 THE STARTING TORQUE AND CURRENT

The theory behind the computation of starting torque and current is presented in chapter 24, paragraph 24.5

$$(I_s)_{S=1} = I_{sc} \sqrt{\left[1 + \frac{t_{es}}{a} \frac{(\omega C |Z_{sc}| \cdot a^2 - \sin \varphi_{sc})}{\cos \varphi_{sc}} \right]^2 + \frac{t_{es}^2}{a^2}} \quad (27.56)$$

$$t_{es} = \frac{T_{es}}{\left(\frac{2p_l}{\omega_l} I_{sc}^2 (R_{r+})_{S=1} \right)} = \frac{K_a \cdot \sin \varphi_{sc}}{a(1 + K_a^2 + 2K_a \cos \varphi_{sc})}$$

$$K_a = \frac{1}{\omega \cdot C \cdot a^2 \cdot |Z_{sc}|}; I_{sc} = \frac{V_s}{2 \cdot |Z_{sc}|}$$

$$\begin{aligned} Z_{sc} &= \sqrt{(R_{sm} + R_{rm})^2 + (X_{sm} + X_{rm})^2} = \\ &= \sqrt{(1.2417 + 3.048)^2 + (4.235 + 3.96)^2} = 9.26\Omega \end{aligned} \quad (27.57)$$

$$\cos \varphi_{sc} = 0.463$$

$$\sin \varphi_{sc} = 0.887$$

$$K_a = \frac{1}{2\pi \cdot 60 \times 25 \times 10^{-6} \times 1.5^2 \times 9.26} = 5.095$$

$$I_{sc} = \frac{115}{2 \times 9.26} = 6.2095A; (R_{r+})_{S=1} = R_{rm} = 3.048\Omega$$

$$T_{sc} = \frac{2 \times 2}{2\pi \cdot 60} \times 6.2095^2 \times 3.048 = 2.495 \text{ Nm}$$

The relative value of starting torque t_{es} is

$$t_{es} = \frac{5.095 \times 0.887}{1.5(1 + 5.095^2 + 2 \times 5.095 \times 0.463)} = 0.095$$

$$T_{es} = t_{es} T_{sc} = 0.095 \times 2.495 = 0.237 \text{ Nm}$$

The rated torque T_{en} , for an alleged rated slip $S_n = 0.06$, would be

$$T_{en} \approx \frac{P_n p_1}{\omega_1 \cdot (1 - S_n)} = \frac{186.5 \times 2}{2\pi \cdot 60 \cdot (1 - 0.06)} = 1.053 \text{ Nm} \quad (27.58)$$

So the starting torque is rather small as the permanent capacitance $C_a = 25 \times 10^{-6} \text{ F}$ is too small.

The starting current (27.56) is

$$(I_s)_{S=1} = 6.2095 \sqrt{\left[1 + \frac{0.095}{1.5} \left(\frac{\frac{1}{5.095} - 0.887}{0.463} \right) \right]^2 + \left(\frac{0.095}{1.5} \right)^2} \approx 5.636 \text{ A}$$

The starting current is not large (the presumed rated source current $I_{sn} = 2.36 \text{ A}$, (Equation 27.23)) but the starting torque is small.

The result is typical for the permanent (single) capacitor IM.

27.8 STEADY STATE PERFORMANCE AROUND RATED POWER

Though the core losses and the stray load losses have not been calculated, the computation of torque and stator currents for various slips for $S = 0.04$ - 0.20 may be performed as developed in chapter 24, paragraph 24.3.

To shorten the presentation we will illustrate this point by calculating the currents and torque for $S = 0.06$.

First the impedances $\underline{Z}_+$, $\underline{Z}_-$ and $\underline{Z}_a$ ((24.11)-(24.14)) are calculated

$$\begin{aligned} \underline{Z}_+ &= R_{sm} + j \cdot X_{sm} + j \frac{X_{mm} \left(jX_{rm} + \frac{R_{rm}}{S} \right)}{\frac{R_{rm}}{S} + j(X_{mm} + X_{rm})} = \\ &= 1.2417 + j \cdot 4.253 + \frac{j \cdot 48.056 \left(j \cdot 3.96 + \frac{3.048}{0.06} \right)}{\frac{3.048}{0.06} + j \cdot (48.056 + 3.96)} = 23.43 + j \cdot 29.583 \end{aligned} \quad (27.59)$$

$$\begin{aligned}\underline{Z}_{-} &= R_{sm} + j \cdot X_{sm} + j \cdot \frac{X_{mm} \left(j \cdot X_{rm} + \frac{R_{rm}}{(2-S)} \right)}{\frac{R_{rm}}{(2-S)} + j \cdot (X_{mm} + X_{rm})} \approx \\ &\approx 1.2417 + j \cdot 4.253 + j \cdot \frac{48.056 \cdot \left(j \cdot 3.96 + \frac{3.048}{(2-0.06)} \right)}{\frac{3.048}{(2-0.06)} + j \cdot (48.056 + 3.96)} \approx 2.7 + j \cdot 7.912\end{aligned}\quad (27.60)$$

$$\begin{aligned}\underline{Z}_a^m &= \frac{1}{2} \left(\frac{R_{sa}}{a^2} - R_{sm} \right) + j \cdot \frac{1}{2} \left(\frac{X_{sa}}{a^2} - X_{sm} \right) - j \cdot \frac{1}{2 \cdot a^2 \cdot \omega \cdot C} = \\ &= \frac{1}{2} \left(\frac{2.435}{1.5^2} - 1.2417 \right) + 0 - \frac{j}{2 \times 1.5^2 \times 2\pi \times 60 \times 25 \times 10^{-6}} = \\ &= -0.08 - j \cdot 23.6 \approx -j \cdot 23.6\end{aligned}$$

The current components $\underline{I}_{m+}$ and $\underline{I}_{m-}$ ((24.16)-(24.17)) are

$$\underline{I}_{m+} = \frac{V_s}{2} \cdot \frac{\left[\left(1 - \frac{j}{a} \right) \underline{Z}_{-} + 2\underline{Z}_a^m \right]}{\underline{Z}_{+} \cdot \underline{Z}_{-} + \underline{Z}_a^m (\underline{Z}_{+} + \underline{Z}_{-})} = 1.808 - j \cdot 2.414 \quad (27.62)$$

$$\underline{I}_{m-} = \frac{V_s}{2} \cdot \frac{\left[\left(1 + \frac{j}{a} \right) \underline{Z}_{+} + 2\underline{Z}_a^m \right]}{\underline{Z}_{+} \cdot \underline{Z}_{-} + \underline{Z}_a^m (\underline{Z}_{+} + \underline{Z}_{-})} = 0.304 - j \cdot 8.197 \times 10^{-3} \quad (27.63)$$

Now the torque components T_{+} and T_{-} are computed from (24.18)-(24.19)

$$T_{e+} = \frac{2p_l}{\omega_l} I_{m+}^2 [R_c(\underline{Z}_{+}) - R_{sm}] \quad (27.64)$$

$$T_{e-} = -\frac{2p_l}{\omega_l} I_{m-}^2 [R_c(\underline{Z}_{-}) - R_{sm}] \quad (27.65)$$

$$T_e = T_{e+} + T_{e-}$$

The source current I_s is

$$\underline{I}_s = \underline{I}_m + \underline{I}_a = \underline{I}_{m+} + \underline{I}_{m-} + j \cdot \frac{(\underline{I}_{m+} - \underline{I}_{m-})}{a} = 3.721 - j \cdot 1.422 \quad (27.66)$$

The source power factor becomes

$$\cos \varphi_1 = \frac{R_e(I_s)}{I_s} = 0.934 \quad (27.67)$$

$$T_{e+} = \frac{2 \times 2}{2\pi \times 60} \times 9.096 \cdot [23.43 - 1.2417] = 2.1425 \text{ Nm}$$

$$T_{e-} = -\frac{2 \times 2}{2\pi \times 60} \times 0.09248 \cdot [2.7 - 1.2417] = -0.967 \times 10^{-3} \text{ Nm}$$

Note The current is about 60 % higher than the presumed rated current (2.36 A) while the torque is twice the rated torque $T_{en} \approx 1 \text{ Nm}$.

It seems that when the slip is reduced gradually, perhaps around 4 % ($S = 0.04$) the current goes down and so does the torque, coming close to the rated value, which corresponds, to the rated power P_n (27.58).

On the other hand, if the slip is gradually increased the breakdown torque region is reached.

To complete the design the computation of core, stray load and mechanical losses is required. Though the computation of losses is traditionally performed as for the three phase IMs, the elliptic travelling field of single phase IM leads to larger losses [5]. We will not follow this aspect here in further detail.

We can now consider the preliminary electromagnetic design finished. Thermal model may than be developed as for the three-phase IM (Chapter 12).

Design trials may now start to meet all design specifications. The complexity of the nonlinear model of the single phase IM makes the task of finding easy ways to meet, say, the starting torque and current, breakdown torque and providing for good efficiency, rather difficult.

This is where the design optimization techniques come into play.

However to cut short the computation time of optimization design, a good preliminary design is useful and so are a few design guidelines based on experience (see [1]).

27.9 GUIDELINES FOR A GOOD DESIGN

- In general a good value for the turn ratio a , lies in the interval between 1.5 to 2.0 except for reversible motion when $a = 1$ (identical stator windings).
- The starting and breakdown torques may be considered proportional to the number of turns of main winding squared.
- The maximum starting torque increases with the turn ratio a .
- The flux densities in various parts of the magnetic circuit are inversely proportional to the number of turns in the main winding, for given source voltage.
- The breakdown torque is almost inversely proportional to the sum $R_{sm} + R_{rm} + X_{sm} + X_{rm}$.
- When changing gradually the number of turns in the main winding, the rated slip varies with W_m^2 .
- The starting torque may be increased, up to a point, in proportion to rotor resistance R_{rm} increase.

- For a given motor, there is a large capacitor C_{ST} which could provide maximum starting torque and another one C_{SA} to provide, again at start, maximum torque/current. A value between C_{ST} and C_{SA} is recommended for best starting performance. For running conditions a smaller capacitor C_a is needed. In permanent capacitor IMs a value C'_a closer to C_a ($C'_a > C_a$) is generally used.
- The torque varies with the square root of stack length. If stack length variation ratio K is accompanied by the number of turns variation by $1/\sqrt{K}$, the torque remains almost unchanged.

27.10 OPTIMIZATION DESIGN ISSUES

Optimization design implies

- A machine model for analysis (as the one described in previous paragraphs);
- Single or multiple objective functions and constraints;
- A vector of initial independent variables (from a preliminary design) are required in nonevolutionary optimization methods.
- A method of optimization (search of new variable vectors until the best objective function value is obtained).

Typical single objective functions F are

F1-maximum efficiency without excessive material cost

F2-minimum material (iron, copper, aluminum, capacitor) cost for an efficiency above a threshold value.

F3-maximum starting torque

F4-minimum global costs (materials plus loss capitalized costs for given duty cycle over the entire life of the motor).

A combination of the above objectives functions could also be used in the optimization process.

A typical variable vector X might contain

1. Outer rotor diameter D_r
2. Stator slot depth h_{st} ;
3. Stator yoke height h_{cs} ;
4. Stator tooth width b_{ts} ;
5. Stack length L ;
6. Airgap length g ;
7. Airgap flux density B_g ;
8. Rotor slot depth h_{rt} ;
9. Rotor tooth width b_{tr} ;
10. Main winding wire size d_m ;
11. Auxiliary winding wire size d_a ;
12. Capacitance C_a ;
13. Effective turns ratio a .

Variables 10-12 vary in steps while the others are continuous.

Typical constraints are

- Starting torque T_{es}

- Starting current $(I_s)_{s=1}$
- Breakdown torque T_{eb}
- Rated power factor $\cos\phi_n$
- Stator winding temperature rise ΔT_m
- Rated slip S_n
- Slot fullness k_{fill}
- Capacitor voltage V_C
- Rotor cage maximum temperature T_{rmax}

Chapter 18 of this book presented in brief quite a few optimization methods. For more information, see Reference 6.

Table 27.1 Design constraints

Constraint	Limit	Standard	F_1	F_2	F_3
Power factor	> 0.92	0.96	0.96	0.95	0.94
Main winding temperature [$^{\circ}\text{C}$]	< 90	86.2	82.2	86.0 (+)	84.8
Max. torque [Nm]	> 1.27	1.29	1.27 (+)	1.28 (+)	1.49
Start torque [Nm]	0.84	0.878	0.876	0.84 (+)	1.06
Start current [A]	< 11.5	10.5	10.9	11.5 (+)	11.5 (+)
Slot fullness $k_{fill} \times \frac{4}{\pi} (\text{to } d^2)$	< 0.8	0.78	0.8 (+)	0.66	0.8 (+)
Capacitor voltage [V]	< 280	263	264	246	261
Efficiency	> 0.826	0.826	0.859	0.828 (+)	0.836

According to most of them the objective function $F(x)$ is augmented with the SUMT, [7]

$$P(X_k, r_k) = F(X_k) + r_k \sum_{j=1}^m \frac{1}{G_j(X_k)} \quad (27.68)$$

where the penalty factor r_k is gradually decreased as the optimization search counter k increases.

There are many search engines which can change the initial variable vector towards a global optimum for $P(X_k, r_k)$ (Chapter 18).

Hooke-Jeeves modified method is a good success example for rather moderate computation time efforts.

Reference 8 presents such an optimization design attempt for a 2 pole, 150 W, 220 V, 50 Hz motor with constraints as shown in Table 27.1.

Efficiency (F_1) and material cost (F_2) evolution during the optimization process is shown in Figure 27.8 a). [8] Stack length evolution during F_1 and F_2 optimization process in Figure 27.8 b) shows an increase before decreasing towards the optimum value.

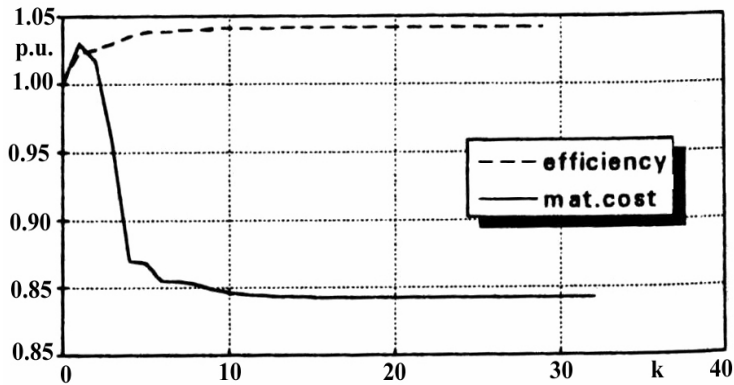
Also, it is interesting to note that criteria F_1 (max. efficiency) and F_2 (minimum material costs) lead to quite different wire sizes both in the main and auxiliary windings (Figure 27.9 a, b).

A 3.4% efficiency increase has been obtained in this particular case. This means about 3.4% less energy input for the same mechanical work.

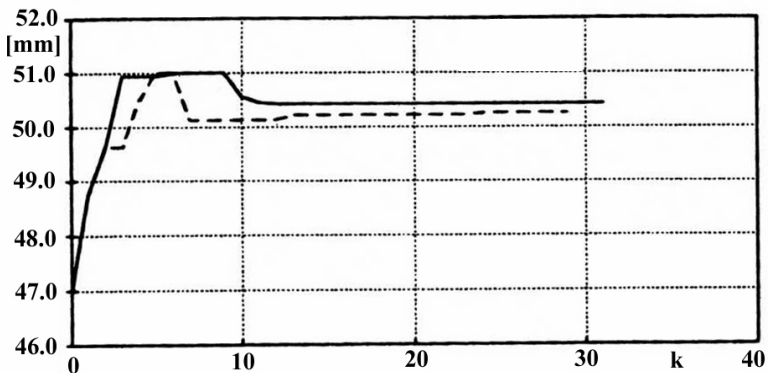
It has to be noted that reducing the core losses by using better core material and thermal treatments (and various methods of stray load loss reduction) could lead to further increases in efficiency.

Though the power/unit of single phase IMs is not large, their number is.

Consequently increases in efficiency of 3% or more have a major impact both on world's energy consumption and on the environment (lower temperature motors, less power from the power plants and thus less pollution).

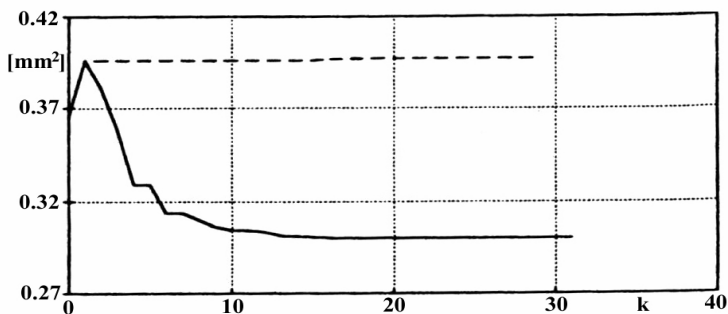


a.)

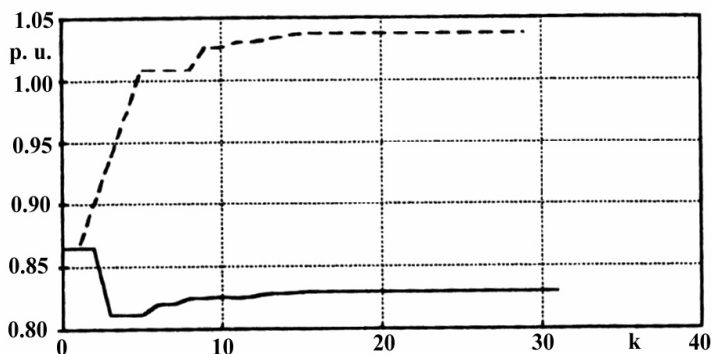


b.)

Figure 27.8 Evolution of the optimization process a.) Efficiency and material costs F_1 and F_2 .
b.) Stack length for F_1 and F_2 .



a.)



b.)

Figure 27.9 Main (a) and auxiliary (b) winding wire sizes evolution during the optimization process

27.11 SUMMARY

- The design of single phase IM tends to be more involved as the magnet field is rather elliptical, in contrast to being circular for three phase IMs.
- The machine utilization factor C_u and the tangential force density (stress) f_t tend to be higher and, respectively, smaller than for three phase IMs. They also depend on the type of single phase IM split phase, dual-capacitor, permanent capacitor, or split-phase capacitor.
- For new specifications, the tangential force density f_t ($0.1-1.5$) N/cm^2 is recommended to initiate the design process. For designs in given standardized frames the machine utilisation factor C_u seems more practical as an initiation constant.
- The number of stator N_s and rotor slots N_r selection goes as for three phase IMs in terms of parasitic torque reduction. The larger the number of slots, the better in terms of performance.

- The stator bore to stator outer diameter ratio D_i/D_o may be chosen as variable in small intervals, increasing with the number of poles above 0.58 for $2p_1 = 2$, 0.65 for $2p_1 = 4$, 0.69 for $2p_1 = 6$ and 0.72 for $2p_1 = 8$. These values correspond to laminations which provide maximum no-load airgap flux density for given stator m.m.f. in three phase IMs [2].
- By choosing the airgap, stator, rotor tooth and core flux densities, the sizing of stator and rotor slotting becomes straightforward.
- The main winding effective number of turns is then calculated by assuming the e. m. f. $E_m/V_s \approx 0.95-0.97$.
- The main and auxiliary windings can be made with identical coils or with graded turns coils (sinusoidal windings). The effective number of turns (or the winding factor) for sinusoidal windings is computed by a special formula. In such a case the geometry of various slots may differ. More so for the split-phase IMs where the auxiliary winding occupies only 33 % of stator periphery and is active only during starting.
- The rotor cage cross section total area may be chosen for start as 35% to 60% of the area of stator slots.
- The turns ratio between auxiliary and main winding $a = 1.5-2.0$, in general. It is equal to unity ($a = 1$) for reversible motors which have identical stator windings.
- The capacitance initial value is chosen for symmetry conditions at an assigned value of rated slip and efficiency (for rated power).
- Once the preliminary sizing is done the resistance and leakage reactance may be calculated. Then, either by refined analytical methods or by FEM (with bare rotor and zero auxiliary winding current), the magnetisation curve $\Psi_m(I_{mm})$ or reactance $X_{mm}(I_{mm})$, is obtained.
- To estimate the starting and steady state running performance (constraints) such as starting torque and current, rated slip, current, efficiency and power factor and breakdown torque, revolving field or cross field models are used.
- The above completes a preliminary electromagnetic design. A thermal model is then used to estimate stator and rotor temperatures.
- Based on such an analysis model an optimization design process may be started. A good initial (preliminary) design is useful in most nonevolutionary optimization methods [5].
- The optimization design (chapter 18) is a constrained nonlinear programming problem. The constraints can be lumped into an augmented objective function by procedures such as SUMT [7]. Better procedures for safe global optimization are currently proposed [9].
- Increases in efficiencies of a few percent can be obtained by optimization design. Given the immense number of single phase IMs, despite the small power/unit, their total power is significant. Consequently, any improvement of efficiency of more than 1-2% is relevant both in energy costs and environmental effects.

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